METHODS OF EVALUATING COMPOUND SINUSOIDAL AND RANDOM VIBRATIONS (METHODS OF EVALUATING VIBRATIONS 2)

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METHODS OF EVALUATING COMPOUND SINUSOIDAL AND RANDOM VIBRATIONS

(Methods of Evaluating Vibrations, 2)

Miwa, T., Yonekawa, Y.*

ABSTRACT. Compound sinusoidal vibrations with several frequency components and random vibrations of a continuous wide frequency band are often encountered in the environmental and industrial investigations. Although a few evaluation methods for these vibrations have been proposed, they are not available to all the cases, and the new method widely applicable has been required.

A ratio scale called vibration greatness in the unit of VG was experimentally determined using the corrected ratio method proposed by W. R. Garner. This corresponds to loudness (sone) in psycho-acoustics. The object was to sum up the VG values of their frequency components. The relations between vibration greatness level (VGL) and vibration greatness (VG) on the whole body and on the hand, for the vertical and the horizontal vibrations, were obtained as in Figure 2 and Table 1.

For the evaluation of the compound sinusoidal vibrations, the vibration acceleration levels (VAL) of their frequency components were converted into VG values (VG_i) by Table 1 through their VGL values using Figures 6 and 9 in Part 1. Next, these VG_i values were summed up in a similar method to one proposed by S. S. Stevens devised for assessment of noise. That is, $VG_T=VG_M+0.3((\Sigma VG_i)-VG_M)$ where VG_M is the maximum value among VG_i values. These calculated values were compared with the observed ones which were sensationally obtained by the comparison of the compound sinusoidal vibration with the vibration of 20 Hz as the standard frequency. These two values are in good agreement with each other below the level of compound vibration of 55 VGL.

For the assessment of random vibrations, threshold and equal sensation curves were determined for the random vibration with 1 and 1/3 octave-band. Their results on the whole body and the hand are shown in Figures 6 and 7. VG values of the frequency components of the random vibrations of continuous wide frequency bands were obtained using these figures and Table 1, and were

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** Numbers in the margin indicate pagination in the foreign text.

summed up with weighting factor 0.3 for the data analyzed by the 1-octave band and 0.13 for 1/3 octave band, in the same method as in the compound sinusoidal vibrations. The calculated values were in good agreement with the observed values in which the random vibration was directly compared with the vibration of 20 Hz.

The sensation difference between vertical and horizontal vibrations was observed for the 1-octave band random vibration. Sensation of the vertical vibration is severer by 10 dB than that of the horizontal vibration at the same vibration level above 16 Hz.

1. Foreword

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A discussion was presented in Report No. 1 concerning a method of evaluating sinusoidal vibrations or vibrations having a single, predominant frequency. Vibration encountered in the actual vibration environment, however, is thought in most instances to be compound sinusoidal vibration or random vibration having a broad, continuous spectrum. In the present report, a study is made of a method of evaluating these types of vibrations.

The K value method [1] and the W_Z value methods are well known as representative conventional methods for evaluating compound sinusoidal waves and random waves. In the former method, K_i is determined for each frequency component and the total K value calculated from . In the latter method, the W_{Zi} value of each frequency component and its frequency of appearance, p_i, are determined and the total W_Z value derived from . In these methods, no study has been made of the effects on the evaluation method of the frequency and interval, the vibration level limits that can be used and the number of frequency components. Additionally, a report on a method of evaluating random vibrations has been seen [3] but no firm conclusions are reached.

Now, when one considers the region of sound, a large number of integrated studies have been made on psychological criteria and loading factors for the evaluation of noise and these have been applied internationally. We thought that this series of methods could be applied to vibrations and conducted the following tests.

2. Vibration Magnitude [4]

The "vibration magnitude level" (unit VGL) determined in Report No. 1 is a distance scale therefore the rules of aggregation do not apply. In order to calculate a total, a ratio criterion, which is a conversion factor, must be applied. This criteria is called "vibration magnitude." It is given that the unit is VG and that 40 VGL are 1 VG. This is a quantity that is analogous to the "magnitude of sound" (sone) which is used for sound. The measurement method used was Garner's corrected ratio method [5]. In this method the subject is asked to make a fractionation judgment and an equisection judgment. The former is a judgment in which the sensation concerning the magnitude of vibrations at a fixed level is divided into 1/2. In the latter, a judgment is required which divides the interval between two fixed vibrations into four parts of subjectively equal magnitude.

Measurements were made of vertical and horizontal vibrations in a seated position and vertical vibrations on the hand and arm with 10 subjects. The posture for the hand and arm system was the same as in the previous report. In preparatory tests on three subjects, the standing position was also tested, but no marked difference was observed between the results for the seated and standing position. Accordingly, the standing position was eliminated.

(i) Experimental Methods

Fractionation Judgment: Vibrations were imposed on the subjects in the order standard level, variable level and rest for four seconds each. This order was repeated and the subjects asked to respond at a vibration level exactly one-half of the standard vibration level sensed. The level changes were in an ascending system and a descending system with five or six measurement levels between 17 and 67 VGL tested. The frequencies were spaced evenly between these limits. The magnitude of the vibration was determined by the following equation.

$$(\text{Vibration magnitude}) = \alpha \cdot \beta^{n} \qquad (1)$$

α - a constant used for moving the values on the VG scale. β - a nominal division ratio, in this test $\beta = 2$. $n=0,-1,-2,-3....$

Equisectional Judgments: To facilitate the judgment of the subjects, the region $17 \sim 67$ VGL was divided into three overlapping portions and the following

tests made on each portion. Initially, the bisection points for the upper and lower levels were measured then each bisection point determined and the section quartered. Finally, the reproducibility of these three division points was tested for each subject. The actual method used was to impose a vibration on the subjects in the descending system in the order upper fixed level vibration, variable level vibration (this level variation was in the descending system), lower fixed level vibration and rest. In the ascending system, the lower and upper fixed level vibrations were reversed and the varied level vibration was in the ascending system. The observed values for the three sections were formed into one curve by proportional allocation. The correlation between the VGL value and log S was determined. This can be expressed by the following equation.

$$\text{Vibration magnitude} = \gamma(S-\delta) \qquad (2)$$ /23

S - VG value with an arbitrary base point. δ - a constant for correcting the arbitrariness of S. γ - a constant having the same meaning as α in Equation (1).

To take the logarithms of Eqs. (1) and (2)

$$\log(S-\delta) = (\log \beta)\cdot n + c \qquad (3)$$

$c \equiv \log \alpha/\gamma$. If an arbitrary value for δ determined by trial and error is substituted, linearity will exist between n and $\log(S-\delta)$. β is determined from that gradient.

The tests were principally made at 20 Hz, however the presence or absence of frequency characteristics at 5, 30 and 60 Hz was also examined.

(ii) Results: δ was determined by trial and error for the values obtained in the above experiments. For example, with 20 Hz whole-body vertical vibration, $\delta = 3.40$, $\beta = 1.71$. I.e. This is lower than the nominal division ratio $\beta = 2$. In the course of the calculations, it was found that there were no differences in the results for whole-body vertical vibration when δ and β were calculated separately for the data for each individual and the VG scale then constructed and when δ and β were calculated for the average values for the 10 subjects and the VG scale then determined. For this reason the latter method was used for the subsequent calculations.

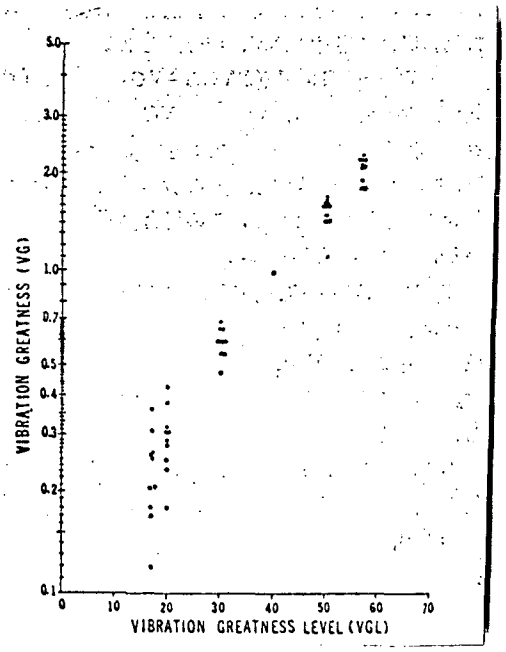

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Figure 1. Dispersion of the relation between VG and VGL values for the whole body vertical vibration individually calculated by the corrected ratio method on 10 subjects at 20 Hz.

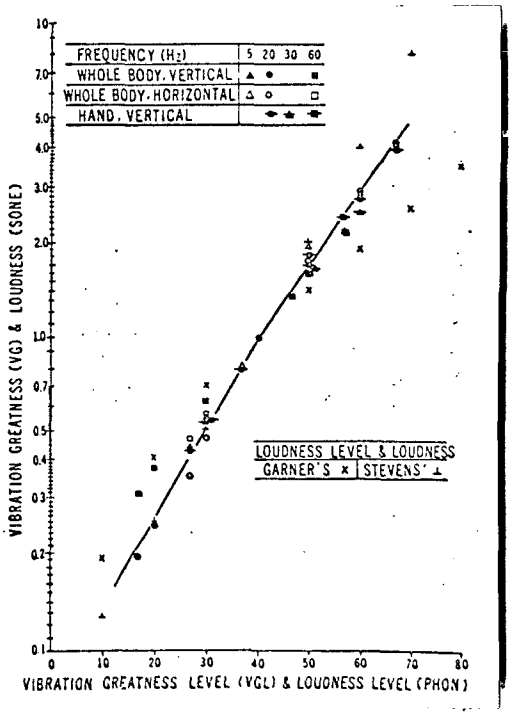
FREQUENCY (Hz) 5 20 30 60
WHOLE BODY, VERTICAL
WHOLE BODY, HORIZONTAL
HAND, VERTICAL
LOUDNESS LEVEL & LOUDNESS
GARNER'S × | STEVENS' ⊥
VIBRATION GREATNESS (VG) & LOUDNESS (SONE)
VIBRATION GREATNESS LEVEL (VGL) & LOUDNESS LEVEL (PHON)

Figure 2. Relation between VG and VGL values on the whole body and the hand vibrations at 5, 20, 30 and 60 Hz. The relation between loudness level and loudness in psychoacoustics obtained by W. R. Garner and S. S. Stevens is also shown in this graph. The unit of VG has been taken as 40 VGL (1 VG = 40 VGL).

Figure 1 shows the correlation between VG and VGL for each individual with 20 Hz whole-body vertical vibration in order to show the levels of disparaties in the present experimental methods. It may be noted that the disparaties increase at the lower levels because it is difficult for the subjects to make a judgment.

The VG-VGL correlation for whole-body vertical and horizontal vibration and hand vertical vibration was plotted on a graph using the average of the measurement on 10 subjects (Figure 2). For purposes of information, Garner's [5] and Stevens' [6] data on loudness levels and loudness are also entered in the Figure 2. Above 1 VG, the VG scale lies between the values of Garner and Stevens but it approximates Stevens' curve below 1 VG.

A study of the frequency dependence of the VG scale was also made and the results for 5, 30 and 60 Hg plotted in the same Figure 2. Virtually no dependence was observed.

This correlation between VG and VGL is represented in the following equations.

$$\text{Below 1 VG, } \log VG = 0.030\ VGL - 1.20 \qquad (4)$$

$$\text{Above 1 VG, } \log VG = 0.023\ VGL - 0.92 \qquad (5)$$

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A table for calculating VG-VGL is given in Table 1.

TABLE 1. CONVERSION BETWEEN VGL AND VG VALUES

VGL	VG	VGL	VG	VGL	VG
5	0.09	20	0.25	35	0.71
6	.10	1	.27	6	.76
7	.11	2	.29	7	.81
8	.11	3	.31	8	.87
9	.12	4	.33	9	.93
10	.13	25	.35	40	1.00
1	.14	6	.38	1	1.05
2	.15	7	.41	2	1.12
3	.16	8	.44	3	1.17
4	.17	9	.47	4	1.23
15	.18	30	.50	45	1.30
6	.20	1	.54	6	1.37
7	.21	2	.58	7	1.45
8	.23	3	.62	8	1.51
9	.24	4	.66	9	1.62

TABLE 1.

VGL	VG	VGL	VG
50	1.70	65	3.75
1	1.78	6	3.94
2	1.88	7	4.17
3	2.00	8	4.40
4	2.10	9	4.59
55	2.20	70	4.90
6	2.35	1	5.14
7	2.45	2	5.43
8	2.57	3	5.73
9	2.70	4	6.03
60	2.88	75	6.37
1	3.05	6	6.72
2	3.23	7	7.08
3	3.40	8	7.46
4	3.58	9	7.87
		80	8.32

3. Compound Sinusoidal Vibrations [7]

Compound vibrations having several predominate frequency components are often encountered in the field. As compound vibration can be readily constructed artificially by mixing sinusoidal wave signals, it is convenient for studying the effects of inhibition between frequencies, vibration levels, the number of component frequencies, etc., on compound sinusoidal vibration evaluation methods. The following experiments were performed. In these tests no consideration was given to phase between the sine wave oscillators. The reason for so doing is that it is not possible to discriminate between phases in sensing vibration [8].

(i) Test Methods: A comparison was made of the magnitude of the sensations between compound sinusoidal vibration and 20 Hz standard sinusoidal vibration using the method of paired comparisons. The subjects subjectively determined equal points with 20 Hz vibration levels being varied in ascending and descending order. As the compound sine wave was a mixture of 2~4 sine waves, they were selected from the central frequencies of the octave bands between 1 and 250 Hz. Vibration was imposed for 10 seconds below 2 Hz and for five seconds above 2 Hz. The VGL values determined empirically were converted to VG values from Table 1. These values were designated as VG_T.

Ten subjects were used and whole-body vertical and horizontal vibration imposed in the seated position. Vertical vibration was imposed on the hand and arm system in the same posture as that given in the previous report.

(ii) Results: In addition to the measured VG_T, the VGL value was determined from the vibration acceleration levels (VAL) for each component frequency in the compound sine wave using Figures 6 and 9 of the previous report. These were then converted to VG values (VG_i). As these VG_i values were simply totaled and no consideration was given to the inhibition effects between contiguous bends, they represent an excess evaluation.

The same is also true in the evaluation of noise. S. S. Stevens [9] has offered a clever weighting method for such cases. Applying this to vibration given Equation (6)

$$VG_T = VG_M + F((\Sigma VG_i) - VG_M) \tag{6}$$

F is a weighting coefficient, VG_M is a maximum value in VG_i. Throughout the above-described experiments, we determined F and also made a study of the effects of band intervals, vibration levels and the number of frequency components on the F value. In these instances, however, VG_T was an observed value. To transform Equation (6)

$$F = (VG_T - VG_M)/((\Sigma VG_i) - VG_M) \tag{7}$$

The Effects of Band Interval on F Value: Measurements of the VGL values were made on compound sine waves of two components (Table 2). The results are shown in Figure 3. The ordinate is the F value while the abscissa is the frequency interval. 0 represents a continuous frequency; 1 represents an interval of one octave band. It is evident from the figure that the F value varies between 0.3 and 1 and is on the order of 0.3 between neighboring frequencies and that the inhibition effect acts, however, it becomes weaker as the frequencies become separated approaching F = 1.

The Effects of Vibration Levels on the F Value: The component vibrations of a compound sinusoidal vibration were mixed at the same VGL values (Table 3), and the VGL value measured for that compound vibration. The results are shown in Table 4. The abscissa is the VGL value estimated by the subjects. The average value is in the center while the upper and lower and the right and left values show the standard deviation. When the compound sinusoidal vibration is

below 55 VGL, F = 0.3, however F begins to decrease above about 55 VGL. The reason for this is thought to be that the dynamic range of vibration sensing is narrow. This trend is common to both whole-body and hand vibration thus, it is believed that F = 0.3 obtains below 55 VGL.

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TABLE 2. THE COMPOUND SINUSOIDAL VIBRATIONS COMPOSED OF TWO COMPONENTS OF DIFFERENT FREQUENCIES IN ORDER TO EXAMINE THE EFFECT OF BAND INTERVAL ON F

Entry	0	1	2	3	4	5	6	7
I	16+32*	8+32*		4+63°	8+250*	1+63*	1+125*	1+250*
II		16+63°		8+125*		4+250°	2+250°	

The sign "+" denotes that two vibrations are compounded at the same level of 40 VGL in the case of sign "*" and 30 VGL in "°". The number indicates the vibration frequency.

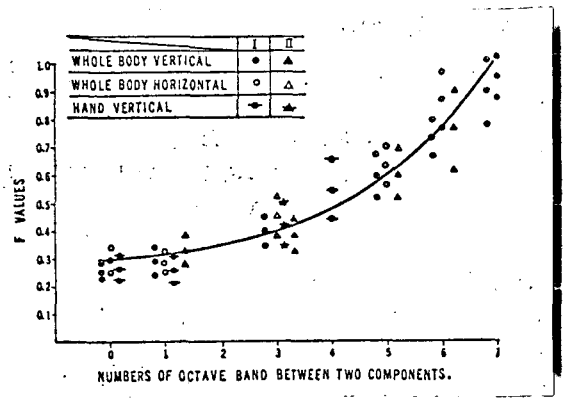

Figure 3. Effect of frequency band between two frequency components of the compound sinusoidal vibrations on F values.

$$F = (VG_{ob} - VG_M)/((\Sigma VG_i) - VG_M)$$

VG_{ob}; observed VG values, VG_i; VG values of their frequency components and VG_M; the maximum VG value among VG_i. Averages and standard deviations on 10 subjects are indicated by three identical symbols arranged lengthwise.

TABLE 3. COMPOUND SINUSOIDAL VIBRATIONS TO EXAMINE THE EFFECT OF LEVEL ON F

Whole body or hand & vibration direction	Frequency components (Hz)	Vibration greatness level of each component (VGL)
Whole body, vertical	16, 32, 63	20, 30, 35, 45
Whole body, horizontal	16, 32	30, 40, 50, 60
Hand, vertical	16, 32	30, 40, 50, 60

Each component of vibration was mixed at the same VGL value shown at the right end column in this table.

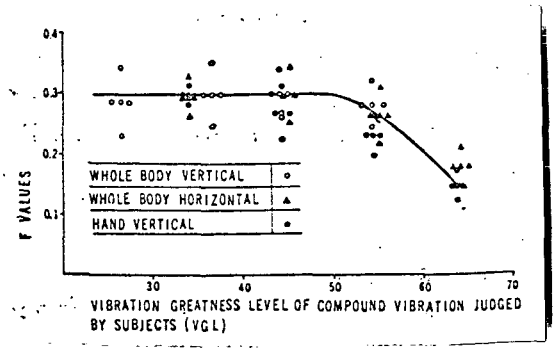

Figure 4. Effect of vibration greatness level of the compound sinusoidal vibrations on F value.

Averages and standard deviations with 10 subjects on F and on judged VGL values are indicated by 5 identical symbols.

TABLE 4. COMPOUND SINUSOIDAL VIBRATIONS TO EXAMINE THE EFFECT OF NUMBER OF THE COMPONENTS ON F AND THEIR OBSERVED F VALUES (MEANS AND STANDARD DEVIATIONS ON 10 SUBJECTS)

Frequency of components (Hz) / Entry	1	8	16	32	63	250		Observed F value
Whole body vertical	25	25			25	25	(VGL)	0.34 ±0.07
	30	27			24	20	(VGL)	0.33 ±0.06
	20	24			27	30	(VGL)	0.29 ±0.06
Whole body horizontal		40	40	40	40		(VGL)	0.30 ±0.04
Hand, vertical		40	40	40	40		(VGL)	0.31 ±0.05

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The Effects of the Number of Frequency Components on the F Values: F varied between 0.3 and 1 with two components. The problem was the degree to which F would contract if the number of components was increased.

The F values were derived for the compound sinusoidal vibration shown in Table 4. As shown on the right-hand side of that table, it was determined that the value contracted to F = 0.3.

From the above results it was determined that VG_T is determined by the following equation for a compound sinusoidal vibration of less than 55 VGL having in its composition a central frequency in the 1~250 Hz octave bands.

$$VG_T = VG_M + 0.3((\Sigma VG_i) - VG_M) \quad (8)$$

4. Random Vibrations [8]

The following tests were made in order to determine a method for evaluating wide band continuous spectrum random vibration. Specifically, equal sensation curves were obtained for octave band random vibrations. Next, the VGL values were measured for a broad band, continuous spectrum random vibration to study the evaluation method given in Eq. (8). A study was also made of 1/3 octave bands.

(i) Experimental Equipment

(a) The Random Signal: A short wave having random time intervals was made by impressing a random voltage signal generated by a zener diode [10] in a flip-flop mode. This signal was passed through 1~1/3 octave band pass filters and recorded in four data recorders as separate octave bands or an arbitrary band signal and used for the tests.

The randomness of these signals was confirmed by measuring the instantaneous amplitude distribution and determining that it had a Gausian distribution on stochastic paper. To supplement this, a study was also made by determining the autocorrelation with an analog correlator.

(b) Generation of Random Vibrations: The vibration platform and the drive section have intrinsic frequency characteristics and these characteristics vary as a function of the body that is placed on the vibrator [11]. In order to generate broad band random vibration accurately, it is necessary to compensate the total characteristics.

An analog type equalizer was used for this purpose. A connection was made from the data recorder, the equalizer, a compensated attentuator (0.1 dB steps) to a switch and then to a power transformer. Additionally, another channel formed from a 20 Hz sine wave oscillator and a compensated attentuator was connected to the other terminal of the switch making it possible to alternate between both channels with the switch at will.

(c) The RMS Value of the Random Vibration: The output from the accelerometer attached to the vibrating stand was led to an acceleration level recorder and recorded in "rms" units. The frequency range of the recorder was 2~200,000 Hz, the pen recording speed was adjusted so that the recorded width was within 3 dB while the average values indicated were rms. When the average values were

difficult to read in low frequency random vibration because the fluctuations of the levels were great, either a statistical analyzer was used or the data recorder was rotated at high speed changing the frequency of the signal. The level was recorded and the rms value determined. The reason for using the rms value was because it is not possible to detect phase differences between signals [8].

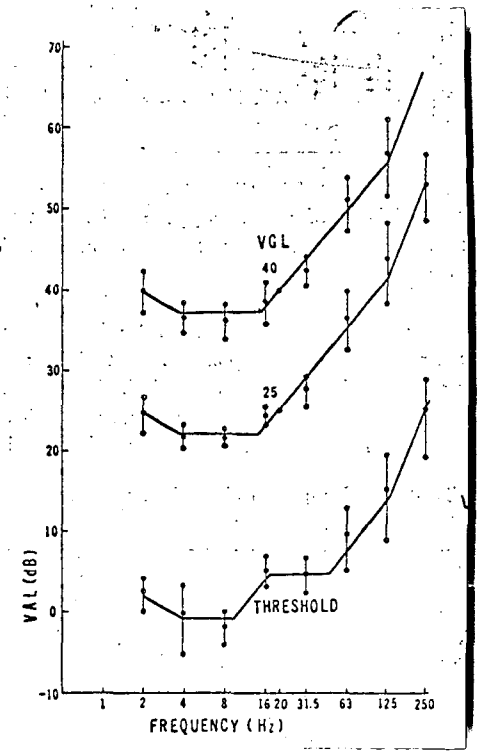

Figure 5. Contours of threshold and equal sensation for the random vibration with 1-octave band for the vertical whole body vibration.

The ordinate denotes the vibration acceleration level (VAL) (VAL = 20 $\log_{10}$ a/a_{ref}; a, rms acceleration value (cm/sec^2), a_{ref} 1 cm/sec^2) and the abscissa the center frequency of 1-octave band. Averages and standard deviations on 10 subjects are indicated by three identical symbols arranged lengthwise.

(ii) Perception Tests

The threshold values and equal sensations were measured for 1~1/3 octave band random vibrations. The method of measuring the threshold was the same as that used for the sine waves in the previous report. For equal sensation, a load was imposed on the subjects in the order random vibration (variable levels), 20 Hz sinusoidal vibration (constant level) and rest. The examiner varied the levels by bracketing and had the subjects respond giving the

equivalent points. Vibration load time was 15 seconds each time and the rest time three seconds.

Measurement of the VGL values for random vibration for the arbitrary band widths was by the same method. Tests were made on all of the subjects for seated position whole-body vertical and horizontal vibration and vertical vibration of the hand and arm, the posture being the same as in the previous report. /27

(iii) Results

Threshold values and equal sensation curves were determined for one octave random vibration with a median frequency of 2~250 Hz. An example of the results is given in Figure 5 which shows the average values, the standard deviation and the approximated curves in whole-body vertical vibration. The standard deviation was ±5 dB. A superimposition of the results for whole-body vertical and horizontal vibration gives the results as shown in Figure 6. Figure 7 was derived for the hand and arm system.

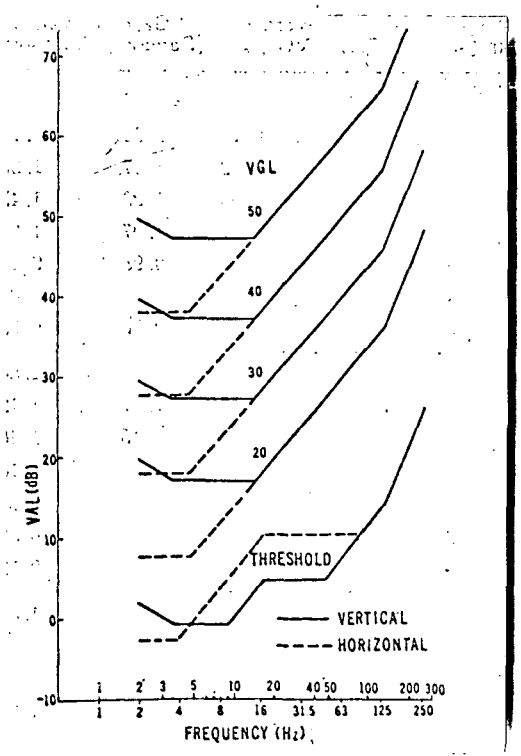

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Figure 6. Synthesized contours of threshold and equal sensation for the random vibration of 1-octave band on the whole body. Results for the whole body vertical and horizontal vibrations are shown in solid and dotted lines, respectively.

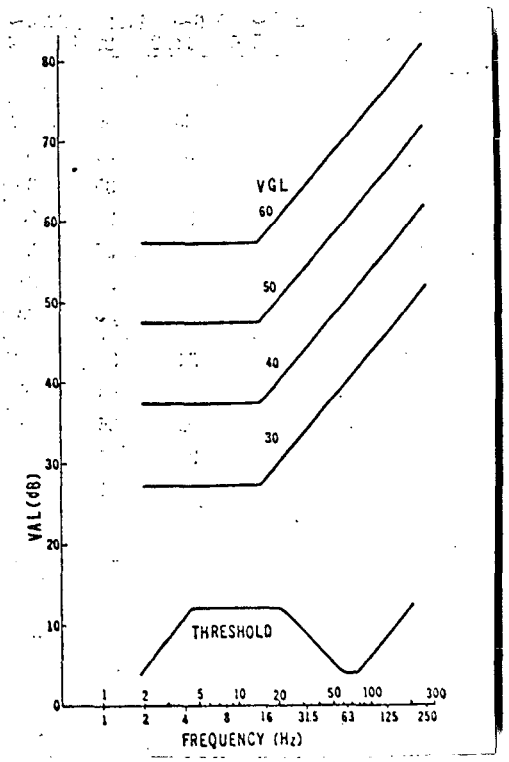

Figure 7. Contours of threshold and equal sensation for the random vibrations of 1-octave band on the hand.

These curves can be used commonly for the evaluation of vertical and horizontal vibrations.

(a) Threshold Values. In Figure 6, the threshold for vertical and horizontal vibration at 8~50 Hz is on the order of 5 dB. Below 5 Hz, the vertical vibration curve is about 4 dB higher, while both curves become superimposed above 80 Hz. Next, a comparison will be made between the threshold values for sinusoidal vibration (Figure 6, previous report) and those for random vibration (Figure 6) in whole-body vibration. In horizontal vibration, the sinusoidal vibration threshold is a maximum of about 10 dB, which is higher than the random vibration threshold of 16~125 Hz. In vertical vibration below 8 Hz, the random vibration threshold had a higher value than the other while at 20~70 Hz the threshold for sinusoidal vibration was a maximum of about 6 dB higher than the other. These differences in threshold values demonstrate that it is easier to recognize random vibration than sinusoidal vibration above 20 Hz.

As compared to the sinusoidal vibration thresholds (Figure 9, previous report) it is characteristic of the hand and arm vibration thresholds that there is a flat section at 5~20 Hz.

TABLE 5. FREQUENCY COMPONENTS OF RANDOM VIBRATIONS ANALYZED BY 1-OCTAVE BANDS PASS FILTER, THEIR OBSERVED VG VALUES ($VG_{T,ob}$) AND CALCULATED VALUES ($VG_{T,R}$ & $VG_{T,S}$)

Center Frequency (Hz)		4	8	16	31.5	63	125	250	Observed average $VG_{T,ob}$	Calculated $VG_{T,R}$ (Random)	Calculated $VG_{r,s}$ (Sinusoidal)
	Band (Hz)	2.8~ 5.6	5.6~ 11.2	11.2~ 22.4	22.4 ~ 45	45~90	90~180	180~ 250			
Entry	VAL										
Whole body vertical	1	30	35	36					1.20±0.14	1.32	1.73
	2	26	31	34	36				1.10±0.12	1.27	1.38
	3		32	35	39				1.07±0.09	1.23	1.45
	4		27	31	36	38			1.98±0.07	1.07	1.20
	5			32	37	38			0.91±0.07	0.98	0.98
	6			28	34	37	40		0.81±0.10	0.87	0.85
	7			28	33	36	39	41	0.74±0.13	0.90	0.85
Whole body horizontal	1	30	34	37					1.65±0.23	1.73	1.84
	2	27	32	33	36				1.52±0.13	1.67	1.72
	3		32	35	36				1.34±0.13	1.45	1.40
	4		27	32	33	36			1.20±0.14	1.16	1.16
	5			31	34	36			1.02±0.11	0.88	0.88
	6			29	32	34	36		0.88±0.08	0.85	0.82
	7			27	29	32	34	37	0.80±0.10	0.77	0.73
Hand vertical	1	32	36	38					1.40±0.11	1.49	1.86
	2	28	33	35	37				1.30±0.12	1.39	1.74
	3		33	35	37				1.30±0.10	1.22	1.48
	4		29	32	34	36			1.09±0.10	1.10	1.27
	5			32	35	37			1.03±0.08	0.94	0.95
	6			30	32	34	37		0.86±0.11	0.89	0.87
	7			28	31	32	35	36	0.80±0.11	0.83	0.78

$VG_{T,R}$ means the VG value calculated from the VGL value obtained by the equal sensation curves for the band random vibration, while $VG_{T,S}$ denotes the VG value calculated from those for the sinusoidal vibration.

(b) Equal Sensation Curves. In Figure 6 the vertical and horizontal curves are superimposed above 15 Hz. No clear difference could be observed in the gradient of the curves as a function of vibration level.

To make a comparison with whole-body sinusoidal vibration, above 15 Hz both random and sinusoidal vibration form curves with the same gradient, without regard to the direction of vibration. The bending point, however, shifts to 125 Hz in random vibration. This bending point was at 60 Hz in sinusoidal vibration. This fact demonstrates that one-octave random vibration is more strongly sensed than sinusoidal vibration in the high frequency band. It can /28
be predicted from the gradient of the equal sensation curve that the vibration components in the vicinity of the cut-off frequency in the low frequency range will feel 3 ~6 dB stronger than those in the vicinity of the central wave length of the band. This is probably the basic factor. To consider low frequencies below 15 Hz next, this also shows that at 12 Hz octave random vibrations are sensed a maximum of about 7 dB weaker than sinusoidal vibrations.

With hand vibration as shown in Figure 7, the curve is simpler than that for sinusoidal vibration and the displacement gradient on the high frequency side is eliminated. To make a comparison with sinusoidal vibration, the same differences as those that appear in whole-body vibration may be observed.

Thus, it was determined that, in the sensation of one octave random vibration and sinusoidal vibration, random vibration is more strongly sensed at high frequencies while sinusoidal vibration is more strongly sensed at low frequencies. In order to determine the appropriateness of this conclusion, a subjective comparison was made of one octave random vibration (variable level) and sinusoidal vibration at the central frequency of each band (fixed level) on 10 subjects at 2~250 Hz.

The results were that, random vibration felt 1 dB stronger at the same levels, at 16 ~63 Hz and 3 dB stronger, at 125 ~250 Hz than sinusoidal vibration. Conversely, below 8 Hz, random vibration felt about 3 dB weaker than sinusoidal vibration. Only in the case of horizontal vibration were the same vibration levels equal. However, the maximum difference of 7 dB which was observed at low frequencies between the random and sinusoidal vibration equal sensation surves did not appear. Additionally, considering standard deviations, the 1 dB difference at 16~63 Hz was disregarded when Figures 6 and 7 were pre- /30
pared.

TABLE 6. FREQUENCY COMPONENT OF RANDOM VIBRATIONS ANALYZED BY 1/3-OCTAVE BAND PASS FILTER AND THEIR CALCULATED VG_T VALUES

Center Frequency		3.15	4	5	6.3	8	10	12.5	16	20	25	31.5	40	50	63	80	100	125	160	200	250	315	Calculated VG_T
Band (Hz)		2.8~3.55	3.55~4.5	4.5~5.6	5.6~7.1	7.1~9.0	9.0~11.2	11.2~14	14~18	18~22.4	22.4~28	28~35.5	35.5~45	45~56	56~71	71~90	90~112	112~140	140~180	180~224	224~280	280~355	
Entry \ VAL																							
Whole body vertical	1	26	27	27	28	32	33	33	33	33													1.39
	2	25	25	23	25	28	30	30	30	30	33	34	35										1.33
	3				25	28	29	30	31	32	34	35	36										1.06
	4				20	23	24	24	26	27	30	31	33	33	34	34							0.98
	5							27	28	28	30	32	33	33	35	35							0.86
	6							23	24	25	28	29	31	32	33	33	34	35	36				0.82
	7							23	24	24	26	28	30	31	32	32	33	35	36	36	36	37	0.85
Whole body horizontal	1	24	25	27	29	30	31	32	33	32													1.63
	2	22	23	24	24	28	28	29	29	29	31	32	31										1.58
	3				26	28	29	30	31	31	32	32	32										1.36
	4				21	23	25	25	27	27	29	29	30	30	31	31							1.10
	5							26	27	28	29	30	30	31	32	32							0.87
	6							25	25	25	27	27	28	29	30	30	30	31	32				0.86
	7							22	23	23	25	26	25	26	28	28	28	29	32	32	33	34	0.75
Hand vertical	1	26	29	30	31	32	33	34	35	34													1.44
	2	23	23	23	26	28	29	29	31	31	33	34	35										1.32
	3				26	27	28	30	31	32	34	34	35										1.17
	4				23	24	25	27	28	28	30	31	32	32	33	34							1.06
	5							27	28	29	31	32	33	33	34	34							0.92
	6							25	26	26	27	29	31	30	32	32	32	34	34				0.89
	7							23	24	24	26	28	28	28	29	29	31	32	33	33	33	33	0.81

(c) 1/3 Octave Random Vibration Equal Sensation Curves [8]. In order to investigate the effects of band width on equality of sensation, a comparison was made of the subjective magnitude of 1/3 octave random vibration and 20 Hz sinusoidal vibrations and measurements made of sensational equality. The frequencies selected for measurement were the central frequencies in the octaves between 2 and 250 Hz. Ten subjects were tested with whole-body vertical and horizontal vibration and hand and arm vibration. The results were in conformity with the results for one octave (Figures 6, 7). These results were thought to be because a comparison was made between random vibration and 20 Hz sinusoidal vibration and that the ability to differentiate the levels between one octave and 1/3 octave is deficient. However, when random vibration at 1/3 octave was analyzed, the figure for the results for an octave was used without modification.

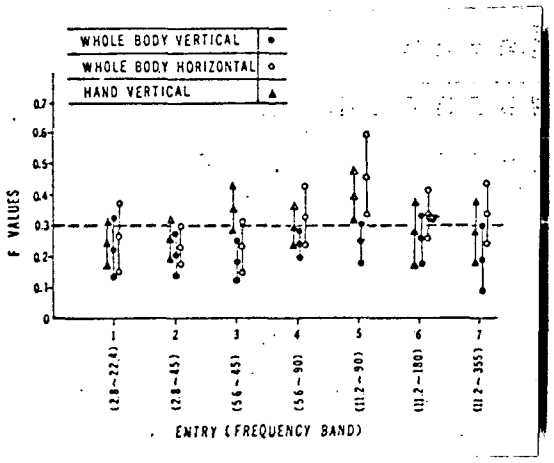

Figure 8. F values of the random vibration of continuous wide frequency band.

The abscissa shows numbers in the entry in Table 5 and the ordinate the F value. Averages and standard deviations of the F values on 10 subjects are indicated by three symbols arranged lengthwise.

(d) The Magnitude of Broad Band Random Vibration. A comparison was made between seven types of random vibrations and sinusoidal vibration at 20 Hz, the VGL values determined and converted into VG values using Table 1 (VG_T). Table 5 shows the octave band components (VAL values) for the seven types of random vibrations, the averages for the observed VG_T values and the standard deviations. These values were then used to calculate F from Figures 6 and 7

and Table 1 using Eq. (7) as was done with the compound sinusoidal vibration. The results were then plotted on a graph to give Figure 8. As indicated, F becomes small for both whole body and hand vibrations when the 5.6 Hz component enters. In the case of whole body horizontal vibrations, the F value is high for vibration under Entry 5. Taking the standard deviation into consideration, however, it was observed that F = 0.3 obtains in random vibration as well.

Next, consideration was given to an analysis of broad band random vibration with a 1/3 octave band-pass filter. As pointed out by S. S. Stevens [9], as well, if it is given that F = 0.3 for one octave, by algebraic calculation, F = 0.13 at 1/3 octaves. Specifically

$$VG_T = VG_{M\cdot 1/3} + 0.13(\Sigma VG_{i\cdot 1/3}) - VG_{M\cdot 1/3}) \qquad (9)$$

The above seven random vibration 1/3, octave analysis values (VAL) were converted in order $VGL_{i\cdot 1/3}$, $VG_{i\cdot 1/3}$ in accordance with Figures 6 and 7 and Table 1, and VG_T calculated by Eq. (9) to give the values shown in Table 6. Comparing these results with the observed VG_T in Table 5, the values agree as VGL values within about ±3 dB. It was thus determined that Eq. (9) obtains within the 1/3 octave bands. A table has been prepared for directly determining the VG value from the 1/3 octave analysis value (VAL) for these calculations [8].

(e) Studies of the Equal Sensation Curves. Up to this point we have been concerned with two types of equal sensation curves. That is to say those for sinusoidal vibration (Figures 6 and 7). A study was then made of the proper use of both curves. That is to say the previously given experimental results for compound sinusoidal vibration were used to calculate VG_T using the two types of equal sensation curves to give the results shown in Table 7. $VG_{T,R}$ shows the valves obtained using the sinusoidal curve. As in the right-hand columns of Table 5, the octave analysis values for random vibration and their $VG_{T,R}$ and $VG_{T,S}$ values are shown. $VG_{T,R}$ for the 1 Hz component of the compound sinusoidal wave was calculated by extrapolation from the random equal sensation curve up to 1 Hz. Throughout both of these results it was evident that, when a low frequency component is present, a compound sinusoidal wave is near $VG_{T,S}$ while with the random vibration, $VG_{T,R}$ is near the measured

value. With high frequency components, there is general agreement between $VG_{T,R}$ and $VG_{T,S}$. That is to say there is a problem only when low frequency components are present.

(f) The Subjective Equation of Horizontal to Vertical Vibration with One Octave Random Vibration. In Figure 6 with whole-body vibration, the equal sensation curves for random vibration above 16 Hz are the same for vertical and horizontal vibration. Like the previous report, however, this illustration is not intended to show mutual correlation. In order to determine this, tests were made making the equalization conversion by the same method as that used for sinusoidal vibration, imposing vibration loads for 15 seconds in seven octave bands between 3.15 and 125 Hz. A hydraulic vibrator was used for only the 3.15 Hz band.

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TABLE 7. FREQUENCY COMPONENTS OF COMPOUND SINUSOIDAL VIBRATIONS, THEIR OBSERVED VALUES ($VG_{T,ob}$) AND CALCULATED VALUES ($VG_{T,R}$, $VG_{T,S}$)

	Frequency (Hz)	1	8	16	31.5	63	250	Observed average $VG_{T,ob}$	Calculated $VG_{T,R}$ (Random)	Calculated $VG_{r,s}$ (Sinusoidal)
	VAL									
Entry										
Whole body vertical	1	19	17			35	58	0.71±0.07	0.74	0.67
	2	24	19			34	53	0.73±0.05	0.62	0,80
	3	14	16			37	63	0.77±0.05	0.95	0.76
	4	24	22	28				0.71±0.05	0.68	0.80
	5			28	34	40		0.78±0.05	0,80	0.80
Whole body horizontal	1		32	38		50	73	1.60±0.12	2.20	1.90
	2			38	44	50		1.44±0.13	1.60	1.60
	3			38	44			1.27±0.05	1.30	1.30
	4		32		44			1.29±0.03	1.30	1.30
	5			28	34			0.67±0.02	0.65	0.65
Hand vertical	1		32	38		50	73	1.63±0.12	2.64	1.90
	2			38	44	50		1.42±0.08	1.60	1.60
	3			48	54			2.09±0.05	2.21	2.21
	4			28	34			0.67±0.01	0,80	0.80
	5		32		44			1.27±0.05	1.21	1.30

The results are shown in Figure 9. The ordinate shows $(VAL)_H - (VAL)_V$ where $(VAL)_V$ is the standard vertical vibration acceleration level and $(VAL)_H$ is the estimated equal sensation horizontal vibration acceleration level. +dB shows that the vertical vibration sensation is stronger than that of horizontal vibration. The abscissa represents the central frequency in each band. Above 16 Hz

the sensation is 10 dB while it drops with the reductions in frequency below 16 Hz. From these results, it was determined that in the equal sensation curves in Figure 6 that 20 VGL vertical vibration is subjectively equal to 30 VGL horizontal vibration.

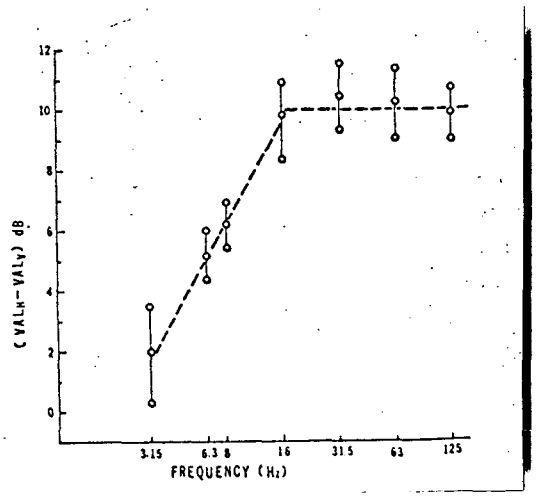

Figure 9. Sensation difference on the whole body between vertical and horizontal random vibrations with 1-octave band.

The ordinate shows $VAL_H - VAL_V$ in dB, where VAL_V is the VAL value of the vertical vibration as the standard and VAL_H the judged VAL value of the horizontal vibration by the subjects. Positive dB in the ordinate means that the sensation of the vertical vibration is severer by the obtained dB than that of the horizontal vibration.

5. Conclusions

Initially, a proportional scale established by the law of aggregation was determined and the "vibration magnitude" (unit VG) was defined. The correlation between the vibration magnitude level (VGL) and VG was taken as in Eqs. (4) and (5).

Next, an evaluation method was studied using the simply handled sinusoidal vibration. Next, a study was made of the effects of frequency intervals, levels and number of frequency components on the weighting coefficient as these effects relate to the VG values of the component frequencies. It was determined that Eq. (8) obtains below 55 VGL.

Next, this addition method was applied to random vibration having a broad spectrum. To do this, the threshold values and the equal sensation curves were determined for 1 ~1/3 octave random vibration. Next, a study was made of

weighting the analytical values for one and 1/3 octave bands for the VG values obtained by using the "vibration magnitude level." It was observed that Eqs. /32
(8) and (9) obtained in all instances.

The question of whether the equal sensation curves for sinusoidal vibration or random vibration should be used when the VGL values for the frequency components in compound sinusoidal vibration and random vibration were being determined was investigated. It was determined that care in the use of both curves is required only when low frequency components below 10 Hz are present.

Additionally, a study was made of the differences in the levels of the equal sensation curves for vertical and horizontal octave random vibration in Figure 6.

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